national accelerator laboratory

EXP-22
August 24, 1972

ACCELERATOR EXPERIMENT--Radial Dependence of Tune and Octupole Moment of the Main Ring at 75 GeV/c

Experimentalists: R. Stiening

Date Performed: 6 July 1972

Measurements by R. Stiening of the radial dependence horizontal and vertical tunes in the main ring at 75 GeV/c on 6 July 1972 are shown on the graph appended to this report. The horizontal tune clearly exhibits curvature; the vertical tune does not. Let us attribute the curvature to the octupole moment of the quadrupoles, and extract an estimate of B'''. Of course, a curvature could also arise in second order from sextupole terms, but that origin of the curvature is unlikely at the high energy at which these measurements were performed.

With the presumption above, the dependence of horizontal tune on radius, or more conveniently, on the fractional deviation of the momentum from that appropriate to the central trajectory, is, for small oscillations:

$$\Delta\nu_x = \frac{1}{4\pi} \sum_{i=1}^{240} \left[\beta_x \left(\frac{1}{2}\,\frac{B'''\ell}{B\rho}\right) x_p^2\right]_i \left(\frac{\delta p}{p}\right)^2$$

where the subscript is indicates that the bracketed quantity is to be evaluated for the $i^{\underline{th}}$ quadrupole of length ℓ as one proceeds around the ring. If β''' has the same absolute value in all the quadrupoles

$$\Delta\nu_x = \frac{1}{4\pi}\left(\frac{B'''\ell_o}{2B\rho}\right)\frac{N}{2}\left[\overline{\left(\beta_x\frac{\ell}{\ell_o}x_p^2\right)}_F - \overline{\left(\beta_x\frac{\ell}{\ell_o}x_p^2\right)}_D\right]\left(\frac{\delta p}{p}\right)^2$$

with $N = 240$ and ℓ_o denoting a standard quad length (7'). The subscript F, here and subsequently, denotes horizontally focusing quadrupoles; similarly, D denotes horizontally defocusing quads. The superposed bars signify averages. For the "White Book" machine, we obtain

$$\Delta\nu_x = \frac{1}{4\pi}\left(\frac{B'''\ell_o}{2B\rho}\right) 120\ (1237\ m^3 - 145\ m^3)\left(\frac{\delta p}{p}\right)^2$$

$$= 1.04 \times 10^4\ m^3\left(\frac{B'''\ell_o}{2B\rho}\right)\left(\frac{\delta p}{p}\right)^2 .$$

From the measurements, $\Delta\nu_x \approx .03$ for $\frac{\delta p}{p} = 0.5\%$. We should probably say $\Delta\nu_x = .03 \pm .01$ as a rough estimate of the uncertainty. Then

$$\frac{B'''\ell_o}{2B\rho} = (0.12 \pm 0.04)\ m^{-3}.$$

If we scale to 200 GeV, we would have

$$B''' = (750 \pm 250)\ kG/m^3.$$

Magnetic measurements made in December, 1970 on "new" quadrupole #7045 suggest $B''' \sim 500\ kG/m^3$, again scaled to 200 GeV.

The curvature of ν_y should be quite small. We have

$$\Delta\nu_y = -\frac{1}{4\pi}\left(\frac{B'''\ell_o}{2B\rho}\right)120\left[360.3\ m^3 - 399.4\ m^3\right] = 0.036\ \Delta\nu_x$$

where the terms are evaluated in the same sequence (F first, D second) as those for $\Delta\nu_x$. It is therefore reasonable that the data for the vertical tune does not show a curvature.

It is interesting to note that the octupole moment suggested by these data favors the resonant extraction scheme presently being pursued. However, at higher energies, the magnet measurements show that the octupole moment of the quadrupoles changes sign.

D. A. Edwards

75 GeV July 6, 1972